Feasibility Study of the Use of Antimatter as a Rocket Propellant

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Introduction

WHEN negative protons (p^-) come into contact with regular protons (p^+), annihilation takes place and matter is converted into energy with the liberation of $2m_0c^2$, equal to 1876 Mev/event. This is equivalent to 3.86×10^{13} Btu/lb or 6.7×10^9 times the energy released by burning a pound of a mixture of hydrogen and oxygen. The velocity of the ejected products (photons) is the speed of light, and the propellant thus becomes a potential one for a photon rocket. The specific impulse is about 30×10^6 sec. These glowing figures must be tempered by the recognition of the very difficult problems of production, storage, and use of such material. These items will be discussed from a feasibility standpoint to see what may be the limiting factors on production rates, storage concentrations and containment times possible in magnetic and electric fields, and propulsion efficiency and weight problems inherent in use.

Production of Antiparticles

Antiprotons

The antiprotons are produced by bombarding suitable targets with high-energy protons from very large accelerators. When these protons strike the protons of target material such as carbon, copper, beryllium, or aluminum, complex nuclear reactions occur, and if the incoming particle energy is high enough, some antiprotons are formed. The threshold for formation is $2\ m_0c^2 = 1.876$ Bev. However, to impart this during the collision the incoming particles must have a minimum energy of 5.6 Bev. It is for this reason that the first accelerator built for antiproton creation was the 6-Bev Bevatron at the Lawrence Radiation Laboratory of the University of California at Berkeley. Other operating accelerators making antiparticles for experimental purposes are the 10-Bev machine at Dubna, Russia; the 30-Bev synchrotron at CERN, Geneva, Switzerland; the 33-Bev alternating gradient synchrotron (AGS) at Brookhaven; and the recent 12.5-Bev zero gradient synchrotron (ZGS) at Argonne.

The production rate is low, but has been increased. The initial beam from the Bevatron in 1955 contained one antiproton every 15 min at a concentration of 1 in 50,000 other particles. This was increased by 1959 to 10/min at a purity of 1 in 10 other particles[1] and on the new AGS is up to 7500/min.[2] This is an improvement of over 1×10^5 times. The process is an inefficient one. The AGS has a proton flux of about 3×10^{11} protons/pulse,[3] and with up to 300 antiprotons/pulse,[2] this is a yield of only about 1 part per billion.

The accelerators have not been optimized for total antiproton production, and the yields undoubtedly can be improved further. This is obviously necessary, since for a large mission requiring 5×10^9 Btu, or 1.75×10^{22} antiprotons (only 0.029 g), this would require about 4.4×10^{12} yr to make at the current rate.

Another factor to bear in mind is that the power from antimatter annihilation has not been previously stored by nature as in the case of that available from fission of uranium or chemical energy available from combustion. All energy has to be fed into the accelerator machines and the inefficiently created antiparticles then stored for subsequent use. Thus, an energetic mission requires extensive prior expenditure of energy on earth.

In spite of the production difficulties, sufficient antiprotons can be made for experimental study, and if the means of storing them could be worked out, development of improved yields would be highly justified.

Positrons

Although the positron-negatron (β^+-β^-) annihilation is much less powerful than that discussed previously, the positrons are easier to obtain for experimental study. Neutron deficient isotopes, either from nuclear fission products or from accelerated proton (p,n) reactions, are emitters. Examples are F^{19} (p,n) Ne^{19}, and C^{12} (p,n) N^{12} reactions where the Ne^{19} and N^{12} give off antielectrons. Some also can be made in a nuclear reactor by means of the (n,γ) reactions. Copper-63 upon absorption of a neutron gives Copper-64, 19% of which undergoes decay to Ni^{64}; likewise, $Zn^{64} \rightarrow Zn^{65}$, which emits 1.7% β^+ particles. These two metals could be used in experimental devices under high vacuum to study antielectron storage and behavior.

Use of Antimatter

Annihilation reaction

The use of antimatter as a propellant is governed largely by the nature of the annihilation reaction. In essence, the p^- and p^+ collision forms pi-mesons (an average of about 4.8/event). These, in a time of about 10^{-8} sec decay into mu-mesons and neutrinos.[1] The mu-mesons, in the order of microseconds, decay into electrons or positrons and neutrinos, and finally the electrons and positrons recombine to give gamma rays. Thus, in a few microseconds, the total rest mass of the $p^- - p^+$ pair degrades to particles with rest mass zero, traveling away with the velocity of light. Direct annihilation into photons may occur, but has not been observed with certainty.

Utilization of gamma-ray energy

In a vehicle, half of the gamma-ray photons from the annihilation will be emitted in the forward direction and must be absorbed and converted to heat to do useful work. They may be used to heat and expel a working fluid such as hydrogen or for generation of electricity to give photon-, arc-, or ion-propulsion. The other half of the annihilation photons will be emitted in the backward direction with a hemispheric ($2\ \pi$) distribution and will give a useful, although dispersed, thrust. The average resultant thrust in the backward direction is $2/\pi$ or 63.6% of the dispersed thrust. This means that 31.8% of the annihilation energy is immediately available as photon thrust.

The 50% of the gamma rays absorbed by the vehicle can be converted to photons in various ways:

1) Part can be changed by scintillators into visible light and be reflected to give useful thrust. The ratio of energy out to energy in for a number of scintillators[4] varies from 0.10–0.15 for NaI to 0.25 for ZnS. Radiation damage (both destruction and darkening) will reduce these values, and many engineering problems would be involved, but it seems possible that 15% might be converted to give 7.5% of the energy from the annihilation. Since the scintillator material would serve as part of necessary shielding, no extra weight penalty would have to be paid.

2) The bulk of the energy absorbed as heat could be converted to electricity both for internal use (pumps, refrigeration, magnetic field generation) and for ion or arc-photon propulsion.

3) Waste heat must be rejected through radiators to outer space. This radiation can be done as infrared at about 1000°K and will also give thrust if the radiating surfaces are suitably designed.

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Under the most optimistic circumstances, the only energy not available for propulsion would be the dispersion loss of primary photons ($50 - 31.8 = 18.2\%$) and the energy used for internal purposes (about 20%). Although this might permit as much as 61.8% of the annihilation energy to provide thrust, a more practical estimate would probably be about 55%.

Mission requirements: volume of antimatter needed

The volume of antimatter to be stored depends on the power demand for any chosen mission. Assuming a Mars orbiter using the equivalent of a 600 kw space power unit reactor (SPUR) power source for a one-year operation,[5] this represents 1.8×10^{10} Btu total energy. Assuming no loss, and assuming that the annihilation energy is 1876 Mev/event, with 55% useful thrust and a stored particle concentration of $3 \times 10^{15}/\text{cm}^3$, a volume of 38.2 cubic meters of antimatter would be required. This volume is large, but if arranged as a torus would be equivalent to a tube of 3.7 ft diam with a torus diameter of 40 ft. This is of the same order of magnitude as some experimental fusion devices.

Thrust

The thrust developed by a photon engine is a function only of the power, and, for an assumed case of 600 kw, with 55% efficiency, it would be

$$F = \frac{1000\ Pe}{c}\ \text{newtons} = \frac{(1000)\ (600)\ (0.55)}{3 \times 10^8\ \text{m/sec}} = 1.1 \times 10^{-3}\ \text{newtons}$$

This is equal to 0.25 mlb thrust. No attempt has been made to estimate the weight of the system, but shielding, containment equipment, and waste heat radiators will be very large, and the thrust-to-weight ratio will be extremely small.

Storage of Antimatter

Possible storage schemes

It is obvious that antimatter must be stored out of contact with normal matter, and this is conceptually possible only with magnetic and/or electric fields. Several methods considered were 1) use of electromagnetic levitation to support solid antihydrogen without wall contact, 2) containing negative protons as separate ions, or 3) containing negative protons and positive electrons as a neutral plasma.

Possible levitation storage of solid antihydrogen

Solid or liquid materials have been supported with no outside contact by using magnetic fields. Electromagnetic levitation[6] occurs when a diverging field is used with a high frequency current (450,000 cps) in a suitably wound coil to induce a current in a conducting sample placed within the coils. The sample is freely suspended and has no contact with the coils. The induced current can heat the sample and, if desired, can cause it to melt, keeping the liquid also freely suspended. If the sample is superconducting at suitably low temperatures, there is no induced current heating because of zero resistance and the solid can be levitated indefinitely.

Conceptually, it should be possible to form antihydrogen from negative protons and positive electrons and, if cooled in some manner to 14°K, the antihydrogen would become solid. This could then be levitated if the antihydrogen were conducting or super conducting. This does not appear probable since ordinary solid hydrogen at 1 atm has a dielectric constant of about 1.20 to 1.28. Formation of metallic hydrogen has been hypothesized,[7,8] and this might have suitable conduction bands to permit levitation. However, such a form of hydrogen does not exist at the highest pressures currently obtainable in the laboratory. Furthermore, super conductivity has never been observed with any single-valence material. Therefore, it is concluded that storage of antihydrogen as a solid is not presently feasible. Levitation, however, would be a satisfactory containment method for any solid or liquid antimatter that was highly conducting or super conducting if it could be formed by any suitable method currently unimaginable.

Possible storage of only ions (protons)

Storage of only protons would be desirable since losses by cyclotron radiation would be small, but in the absence of the oppositely charged electrons a space charge of impossible voltage would result at any desirable concentration and volume. It is a corollary of this that electric fields cannot be used for containment. Since the field would attract one charge and repel the other, it could not act to store a neutral plasma.

Storage as plasma

The storage of antimatter in magnetic fields can draw on the considerable research that has gone into confinement methods for fusion work. The goal of the latter, which has not yet been attained, has been to hold a deuterium plasma for times (t) up to 0.1 sec and at concentrations (n) in the neighborhood of 10^{15} particles/cm^3 so that the product of (n) (t) is at least 10^{14} particles-sec/cm^3. It is also necessary to heat the deuterium plasma to the ignition temperature of 36 Kev (4.1×10^{8}°K). Antimatter, because of greater liberation of energy, could be useful at lower concentrations, should not require the high temperature needed for fusion, and might be kept at lower magnetic field intensities. It would, however, require a containment time many orders of magnitude greater and could not use a pulsed system.

Magnetic fields

The magnetic fields required depend on the particle concentration and are from 90 to 160 kgauss[9,10] at 1 to 3×10^{15} particles/cm^3. These fields are difficult to attain in steady state with conventional magnets. Until recently, the most powerful was 152 kgauss, but a 255-kgauss field can now be produced at the National Magnet Laboratory.[11] However, the bore diameter of the magnet is only $2\frac{1}{8}$ in., and 10.5 Mw of power are required for operation. Developments in superconductivity magnets give promise of field strengths up to 200–340 kgauss with Nb_3Sn and up to 500 kgauss with V_3Ga and are potentially capable of large volumes at somewhat lower field strengths. At the same time, power requirements are negligible (although refrigeration power is a definite factor). It is felt that these new discoveries will be potentially valuable for fusion and for the application considered here.

Storage devices

There are many detailed descriptions of magnetic field storage devices that have been studied for fusion, including Stellarators, magnetic mirrors, and storage rings. None has been successful in all respects. Lauer and coworkers[12] have studied mirror retention using positrons (β^+) from radioactive Ne^{19}. They observed a mean containment time of greater than 10 sec. O'Neill and Wood[13] have proposed a ring for protons with an estimated storage time of up to 30 hr. The Frascati storage ring in Italy[14] has been designed for holding 250 mev electrons and positrons for up to 250 hr. However, in all of these the concentrations of particles are very low. Many problems of storage instability exist and these cannot be detailed here, but the following sections point out some of the important controlling ones.

Coulomb scattering

Because of the very long times of containment necessary, coulomb scattering appears to be a major limitation and requires very high energy for the stored particles. When similarly charged particles, such as protons, approach each

Table 1 Relaxation time for various energies

Particle energy	Relaxation time, sec
10 ev	1.42×10^{-6}
36 ev	0.31
300 Kev	7.4
170 Mev	10^5 (27.7 hr)
3 Bev	7.4×10^6 (86 days)

other, they are repelled and can be scattered through large angles (90° or more) in a single encounter and thus escape from the containment device. The cross section σ_c for this close collision is given by (see Ref. 10, p. 88)

$$\sigma_c = e^4/4\,W^2 = 1.6 \times 10^{-14}\ \mathrm{cm}^2/W^2$$

where W is the center of mass (or relative) kinetic energy of the interacting particles in electron volts. If t is the relaxation time or time per scatter to produce a substantial change in concentration, then $t = 1/n\,\sigma_c v$, where n is the concentration (assume 10^{15} particles per cm^3), and v is the particle speed. Since v is proportional to $W^{1/2}$, and since at thermal temperatures v is 22×10^5 cm/sec and W is 0.025 ev, it can be shown that $v = 13.9 \times 10^5\ W^{1/2}$, and therefore $t = 4.5 \times 10^{-8}\ W^{1/2}$. The relaxation time for various energies is shown in Table 1. The results indicate that retention, even for a day, would require very high energy in the hundreds of Mev.

Electron energy loss

At the high energies required for the antiprotons, the accompanying positive electrons would lose energy very rapidly by cyclotron radiation (proportional to the square of the energy) and, to a lesser extent, by bremsstrahlung radiation (proportional to the square root of the energy). This loss would have to be made up in effect by "cannibalizing" energy from the stored antiprotons and in a very short time no energy would be available for propulsion.

The significant fact is that at the high energies needed to reduce coulomb scattering of the antiprotons and permit their possible storage the concurrent losses of energy due to the positive electron radiation become unbearable.

Conclusions

Although antimatter would be a desirable propellant, its preparation in useful quantities is not currently possible, and no way can be visualized at present for storage of antiparticles. The high energies necessary to reduce coulomb scattering cause unbearable energy losses due to electron radiation. There appears to be no way out of this dilemma in the light of current understanding. Unforeseen basic developments in the next 8–10 years might alter the picture, but this is pure speculation. Future developments in basic plasma and fusion research should be watched, but no further feasibility work is justified for the present.

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Ergodic Theory and Certain Stochastic Satellite Problems

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I. Introduction

THE particular class of problems which motivated this report are as follows: We are given a certain initial configuration of satellites in their orbits. We are also given a particular "geometric"† event E of interest in connection with the performance of this system. A typical event E might be "the subsatellite point of at least one of the satellites in the system falls within a designated area on the surface of the earth," or "all of the satellites are simultaneously clustering around their respective southernmost points." We wish to define and calculate the probability of events of this nature and determine how this probability varies as a function of the initial condition. Transients, however, are not discussed. The results proven below were the consequence of an attempt to generalize and derive rigorously certain similar statements to be found on pp. 149 and 150 of Ref. 6. The bulk of the mathematical results utilized belong to those disciplines usually called "ergodic theory" and "diophantine approximation." Certain critical results referred to in the text are stated in the appendix for reference.

II. Mathematical Preliminaries

Let us assume we are given a system of n satellites in n (not necessarily distinct) orbit planes about a spherical nonrotating earth. Each of the j ($j = 1, \ldots, n$) satellites at time zero is assumed to be in a circular orbit of altitude h_j, have orbit inclination i_j, and have ascending nodal location Ω_j. The angular rate ω_j of the jth satellite is presumed constant. Let $\theta_j^*(t)$ be the true anomaly of satellite j at time t measured in the sense of motion around the orbit from the ascending node. We shall agree to use units of revolutions per unit time for ω_j and define $\theta_j(t)$ as $\theta_j^*(t)$ modulo I. More precisely, for any real number α, let $[\alpha]$ be the largest integer in α, so that $[\alpha] \leq \alpha < [\alpha] + 1$. Let $\{\alpha\}$ be the fractional part of α, i.e., $\{\alpha\} \equiv \alpha - [\alpha]$. Then $0 \leq \{\alpha\} < 1$. In general, we shall use the expressions "α modulo 1" and "$\{\alpha\}$" interchangeably.

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† The meaning of the adjective "geometric" will be explained more precisely in the following.